

# MAT 800 (Topics in Analysis)

Fall 2013

*TITLE AND COURSE INFORMATION*

## Sobolev Mappings and Variational Integrals in Geometric Function Theory with applied disciplines in mind

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**Office Hours:** Tuesday 1:00 - 2:00 pm  
Thursday 1:00 – 2:00 pm  
At other times by appointment

**Classes:** T Th 2:00 - 3:20. Heroy Auditorium (Geology Building)

**Text:** Beamer presentations and notes will be provided during the course.

**Prerequisites:** I will assume basic knowledge of topology, measure theory, integration, analytic functions and basics in functional analysis.

**Course Description:** We will cover the following topics:

1. *Null Lagrangians and Polyconvex Variational Integrals.* In Nonlinear Elasticity it is almost axiomatic that the stored energy functions must satisfy Morrey's quasiconvexity condition. For, it helps to describe the passage from microscales to macroscales (a thoughtful view by J. Ball). On the other hand, polyconvexity is the only practical way to construct quasiconvex energy integrals.
2. *Energy-Minimal Deformations.* Theoretical prediction of failure of bodies caused by interpenetration of matter is a good motivation that should appeal to mathematical analysts and researchers in the engineering fields.
3. *Invertibility versus Lagrange Equation for the Traction Free Energy-Minimal Deformations.*

Analysis of harmonic homeomorphisms between planar domains (thin plates and films) will lead us to the Hopf Laplace equation, and a charming connection with minimal surfaces.

4. ***Limits of Sobolev Homeomorphisms and Approximation with Diffeomorphisms.*** As we seek greater knowledge about the energy-minimal deformations the weak and strong limits of Sobolev homeomorphisms become ever more quintessential. We arrive at the monotone Sobolev mappings. Topology comes into play.
5. ***n-Harmonic Hyperelasticity.*** Moving to a realm of energy-minimal homeomorphisms in higher dimensions has its own challenges, due to several factors. The ultimate goal is to secure injectivity of the energy-minimal deformations, prompted by the principle of no interpenetration of matter in Nonlinear Elasticity.
6. ***The p-Harmonic Equations.*** These are prototypes of many nonlinear elliptic systems of partial differential equations. We devote some time to an introduction and overview of the p-harmonic analysis.

Every effort will be made to reduce to a minimum the technical complications and mathematical requirements. No prior exposure to these topics is assumed. You are invited to pursue the exercises given occasionally during the course. Some of them are far from being routine.

**Note:** Students who may need special consideration because of any sort of disability should make an appointment to see me during office hours. This should be done very early in the semester to make suitable accommodations.