MAT 830 Fall 2016 – Syllabus

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Office Hours: Monday 11:45am -12:40am, Tuesday 3:30pm-4:30pm, Wednesday 1:55pm-2:50pm and by appointment. (Any changes will be announced in class and via email.)

Hopf Algebras, Actions and Invariants.

As Rotman points out in his graduate algebra text, Hopf algebras have become ubiquitous in mathematics. The goal here is to provide a gentle introduction which will convince people that they have in fact seen examples and applications before.

Organization:

The pace and level of detail, including which proofs are covered, will be determined by students' backgrounds. We will need to draw from commutative algebra and algebraic geometry. Since not everyone will have seen these topics, the necessary material will be sketched without proofs and references will be given.

Requirements:

Homework problems will be listed in class but not collected. Solutions provided in some cases. You are expected to attend and ask questions either in class or office hours when you do not understand something.

References

S. Montgomery Fixed rings of finite automorphism groups of associative rings, LNM Springer 1980
S. Montgomery Hopf Algebras and their Actions on Rings, LNM
D. Passman The algebraic structure of group rings, Wiley
D. Passman Infinite crossed products, Academic Press
S. Dascalescu, C. Nastasescu, S. Raianu Hopf Algebras - An Introduction, Monographs and Textbooks in Pure and Applied Mathematics
J.E. Humphreys Linear Algebraic Groups, GTM Springer
J.E. Humphreys Lie Algebras and their Enveloping Algebras, GTM Springer
N. Jacobson Algebra II

Topics:

1. Galois Theory of field extensions revisited. After introducing the theory of central simple algebras we will see that the Galois theory of field extensions (in all characteristics) can be extracted with little effort. In addition we can prove some results not included in a first course on Galois theory.

2. Hopf algebras. The easiest example is the group algebra, which plays a large role in topic 1 above. A Hopf algebra is one that can “act” on another algebra in a natural way. We will set up the basic theory and constructions including examples before turning to important special cases. In the finite dimensional case, the subject is still motivated to some extent by a list of conjectures made by Kaplansky. The state of progress on these will be surveyed.

3. Algebraic groups. These are groups whose underlying set is an algebraic variety, where the product and inverse operations are morphisms of varieties. A more algebraic approach is to view these as the set of algebra maps from an affine (finitely generated as an algebra), commutative Hopf algebra to the field k.

4. Invariants of subgroups of $GL_k(V)$ acting on the coordinate ring of V. Essentially this means the subalgebra of a polynomial ring $R[X_1,\ldots,X_n]$ consisting of the polynomials fixed pointwise by every element of a group G of k -linear automorphisms. These automorphisms preserve homogeneous polynomials, or equivalently, each automorphism in G restricts to a k-isomorphism of the span of $X_1,\ldots,X_n$.

5. Lie algebras and their enveloping algebras. These form an important class of examples of Hopf algebras.

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